CACTUS

Hartley Hyde • cactus.pages@internode.on.net

The ubiquitous parabola

Every so often someone who once suffered a poor learning experience, challenges the inclusion of key topics in the curriculum. When an author and a British politician both questioned the teaching of quadratic equations the AAMT email list members responded admirably. The list provides a friendly, free and often sympathetic forum for this type of debate. If you still have not joined the list, contact the AAMT office today.

I liked the thread seeking 101 uses for a quadratic equation. A study of quadratics is important because a quadratic is the simplest non-linear function. The importance of non-linear functions in regression modelling will become more apparent as a new generation of students, who have learned to use statistics and technology, take their place in the workforce.

Even back-to-basics folk have to admit that quadratic factorisation provides a late opportunity to drill simple number skills. It was only yesterday that I watched a Year 11 student struggle to factorise $x^2 - 5x - 36$ because she had been allowed to forget that 4 and 9 are factors of 36.

A Tammock solar-powered grill

Less easy to defend has been the teaching of the parabola as an example of a conic section, and sadly, a study of conics has often been a casualty to the constant erosion of our curriculum time. Conics problems provide an ideal opportunity to show how simple algebra can be used to model geometry. The topic is supported by so many interesting applications ranging from the orbits of objects in space to the design of loud speaker systems.

Although the earth's gravitational field is radial, it is so large that small sections, such as the gravitational field within a room, skate park or cricket pitch can be regarded as parallel. This means that any object in free fall, be it a piece of chalk, a skate board rider or a ball, will follow a parabolic trajectory.

Parabolic shapes are easy to find. Many homes outside the cable area have parabolic antennas to receive television transmissions from a satellite far out in space. Parabolic extrusions are used as reflectors behind fluorescent tubes and solar heated water pipes. Parabolic reflectors are used to build solar ovens, searchlights and radio telescopes.

The following two pages are presented as work sheets. The first is designed for use with *Cabri Geometry* to study the parabola as a conic section. The second presents a similar construction for a circular paraboloid using *Cabri 3D*. Both models show clearly that the parabolic locus will reflect incoming rays, that are parallel to the line of symmetry, back through the focus.

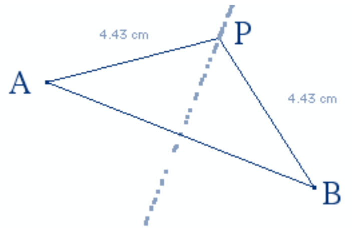
These pages also illustrate a use of the two *Cabri* products to study a 2D object before considering its 3D equivalent. *Cabri 3D V2* now allows us to insert measurements, equations and coordinates. It also allows us to trace the trajectory of a point or curve.

Pricing structures for the new version of *Cabri 3D* are available from the AAMT office.

Calculator And Computer Technology User Service

The parabola

A work sheet for use with Cabri Geometry



A locus is a set of points that obey a rule. For example, if we trace all of the points P which are the same distance from two fixed points A and B, as shown above, we find that all the possible positions of P lie on the perpendicular bisector of AB.

During this lesson we will find the locus of all the points that are equi-distant from a line (called the directrix) and a point (the focus).

Launch *Cabri Geometry II plus* or later.

At the top of the screen is a toolbar. Check the options available for each tool. Write down where to find the various tools highlighted on this sheet.

From the right-most tool, which the user manual calls **attributes**, choose **show axes**.

Use the **point** tool to place D at (–8, 0).

Use the fifth tool **[constructions]** to draw a line **perpendicular** to the *x*-axis through the point D. This is the directrix.

Place a **point** A on the directrix.

Draw a line **perpendicular** to the directrix through the point A; i.e., parallel to the *x*-axis.

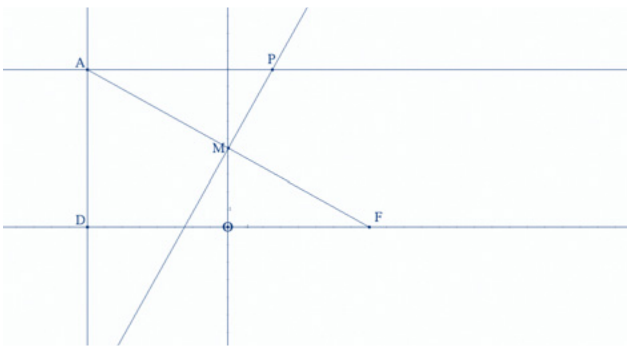
Place a **point** F at (8,0).

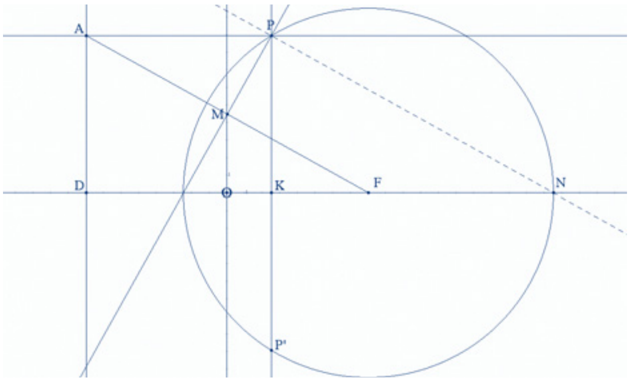
Choose **[lines] segment** and join the points A and F with a line segment.

Choose **[constructions] perpendicular bisector** and click on the line segment AB.

Choose **[point] intersection points**, then click on the segment AB and then click on the perpendicular bisector. A new point will appear where these lines cross. Label it M.

Similarly, find and label the point P where the lines AP and MP intersect (see below).





Because AP is perpendicular to DA, the length AP is the distance of the point P from the directrix. Since P is on the perpendicular bisector of AF, it must be equi-distant from the directrix DA and the focus F. Draw a circle with centre F through P. The radius is equal to the distance of P from the focus. This is the same as the distance of P from DA. Therefore the point P is one of the points on the locus we are trying to construct.

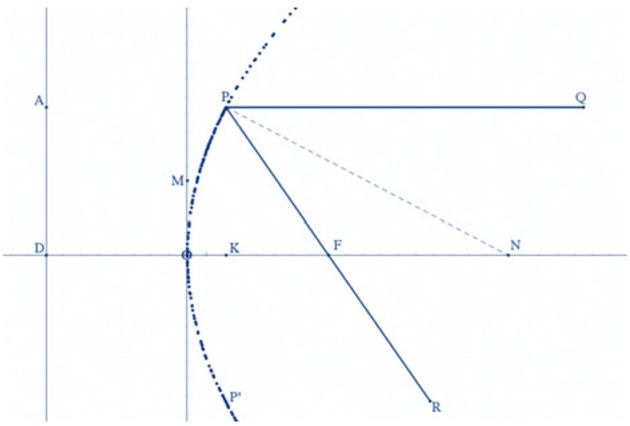
Use **[constructions] parallel** to find another point, P', which is the same distance from the directrix and from the focus. Find the point where PP' crosses DF and label it K.

Find the **intersect point** of PM and DF. Label it B. Draw a line PN **perpendicular** to the line MP at the point P. Choose the **pointer** and use it to move the point A up and down the directrix. Study what happens to the points M, B, N and P as you do this.

Choose the **Trace** tool and click on the point P. Move the point A up and down the directrix. As you do this, the point P will trace the locus of points along a parabola with tangent MP and normal PN. Repeat this process using the **Locus** tool.

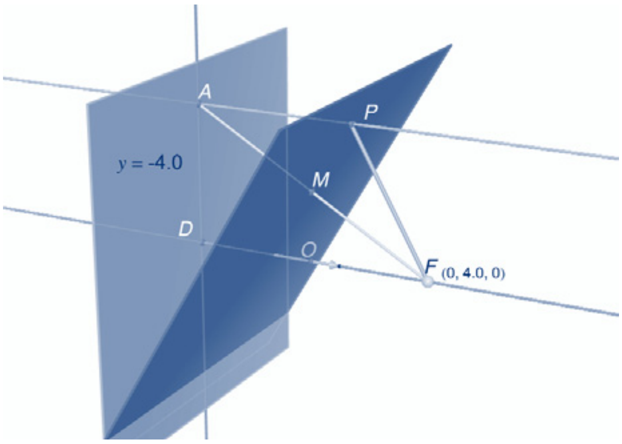
Choose a point Q on the line AP. Now reflect the segment QP in the normal PN. Move the point A. If QP is an incoming ray of light, notice that is always reflected from the parabola, back through the focus.

Clean up by hiding the construction lines.



The Paraboloid

An extension activity following The Parabola



Open the application *Cabri 3D Version 2*. Familiarise yourself with the toolbar options and write down where to find the tools highlighted on this sheet.

Use the control/command M key to delete the unit vectors *i* (red) and *k* (blue). Choose the **line** tool and click on the *j* vector (green).

Place a **point** D toward the left end of the line and a **point** F toward the right end. Use the **coordinates** tool to mark their position. Adjust the position of the points so that D is at (0,–4.0,0) and F is at (0,4.0,0).

Construct a plane **perpendicular** to DF through the point D. Mark its equation y=–4.

Construct a line **perpendicular** to the plane z=0 through the point D.

Place a **point** A on this vertical line.

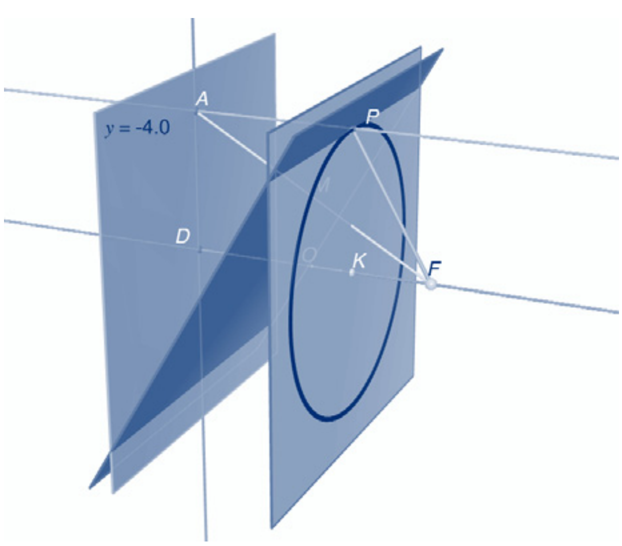
Hide the horizontal plane z = 0.

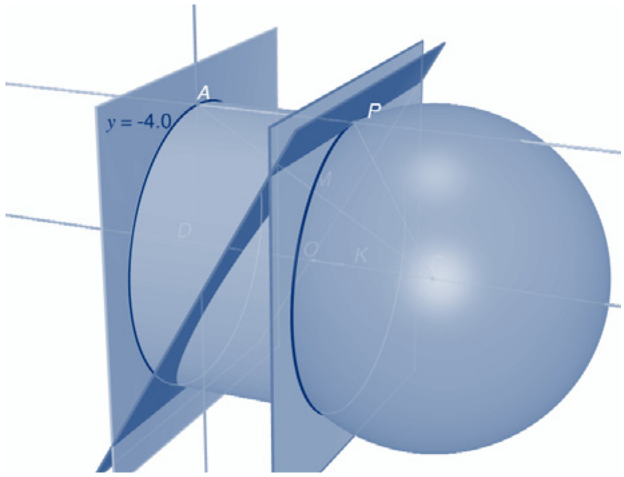
Construct the **line segment** AF.

Construct the **perpendicular bisector** of AF and mark the point M where it cuts AF.

Draw a line through A **parallel** to DF to cut the perpendicular bisector plane at P.

Construct a plane **perpendicular** to DF through the point P to cut DF at K.





Draw a **cylinder** about the **segment** DK, through the point P. Draw the circles where the cylinder **intersects** the vertical planes.

Draw a **sphere** centre F passing through P.

Because MP is the perpendicular bisector of AF, the radius of the sphere FP is equal to the length of the cylinder AP. Thus P is on the locus of points equidistant from the directrix plane through D and the focus point F. The cylinder and sphere show that all points on the circle through P are also on this locus.

Hide the cylinder and the sphere.

Choose **trajectory** and click on the circle through P. Then click on the **manipulation** tool and move the point A up and down the line DA. As you move A, the program will attempt to trace a sample of images of the locus circle. The pattern of images generated will depend on how slowly and smoothly you can move the mouse while you do this.

Use the **active view** to set the figure rotating and you will recognise that the locus is a paraboloid and that the perpendicular bisector plane is a tangent to the paraboloid.

Construct a plane through P **perpendicular** to MP. Place a line segment QP on AP and reflect the segment in the plane. The reflection passes through the focus.

Clean up and colour to taste.